Q1 [Alexander and Sadiku, 2009, Q5.41]

Monday, July 1, 2013 4:02 PM

We want $v_o = -\frac{1}{4}(v_1 + v_2 + v_3 + v_4)$

$$= -\left(\frac{1}{4}v_1 + \frac{1}{4}v_2 + \frac{1}{4}v_3 + \frac{1}{4}v_4\right). \quad \ldots (1)$$

Using summing amplifier with four inputs, we get

$$v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3 + \frac{R_f}{R_4}v_4\right). \quad \ldots (2)$$

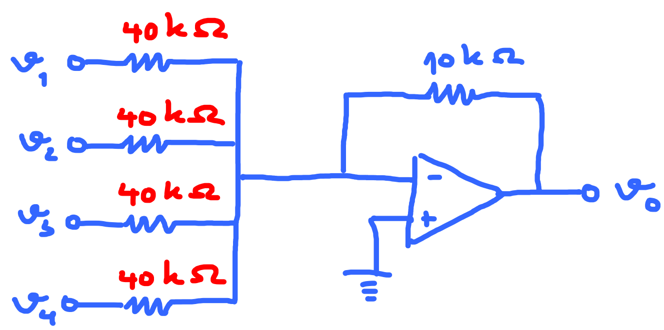
By comparing (1) and (2), we see that we need

$$\frac{R_f}{R_1} = \frac{R_f}{R_2} = \frac{R_f}{R_3} = \frac{R_f}{R_4} = \frac{1}{4}$$

With $R_f = 10\,k\Omega$, we have

$$R_1 = R_2 = R_3 = R_4 = 4 \times 10\,k\Omega = 40\,k\Omega$$

Therefore, the specified averaging amplifier can be built from a summing amplifier:



Q2 [Alexander and Sadiku, 2009, Q5.47]

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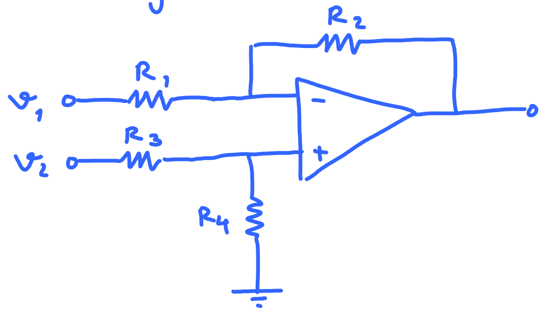
First, this problem is very easy when we realize that the given circuit is a difference amplifier.

Recall the circuit diagram of a difference amp:



$$v_o = -Av_1 + \frac{1+A}{1+B} v_2$$

where $A = \frac{R_2}{R_1}$ and

$B = \frac{R_3}{R_4}$

Matching the above diagram with the provided figure in the problem, we see that

$$R_1 = 2k, \; R_2 = 30k, \; R_3 = 2k, \; R_4 = 20k$$

Therefore,

$$A = \frac{R_2}{R_1} = 15 \quad \text{and} \quad B = \frac{R_3}{R_4} = \frac{1}{10}$$

which give

$$v_o = -15 v_1 + \frac{1+15}{1+\frac{1}{10}} v_2 = -15 + \frac{160}{11} \times 2 \approx 14.09 \; ✓$$

Of course, one may work on this problem from scratch using the two main characteristics of ideal op amp along with applications of KCL or the voltage divider formula. (This was exactly what we did in class to derive the expression for v_o; so, we do not repeat the whole process again here.)

Q3 [Alexander and Sadiku, 2009, Q5.29]

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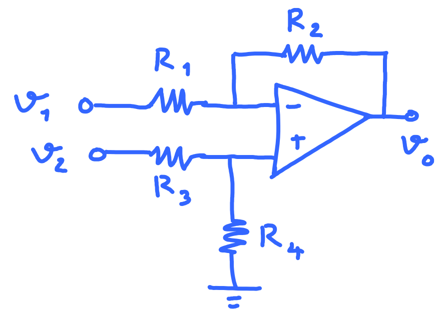
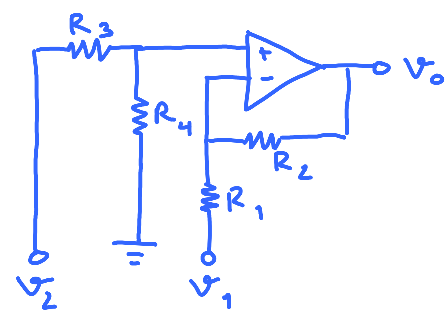
First, this problem is very easy when we realize that the given circuit is a difference amplifier. To see this, compare the two circuits below.

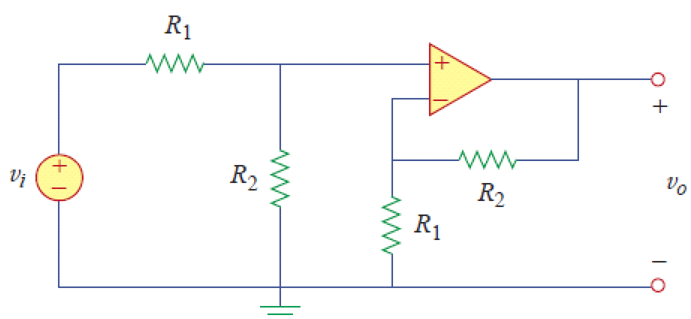


compact form

$\equiv$



alternative form



The given circuit has

$$v_2 = v_i \quad , \quad v_1 = 0 ,$$

$$R_3 = R_1 \quad , \quad R_4 = R_2 .$$

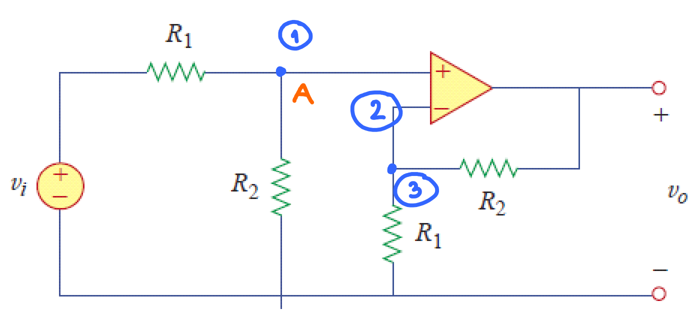
$$A = \frac{R_2}{R_1} \quad \text{and} \quad B = \frac{R_3}{R_4} = \frac{R_1}{R_2} = \frac{1}{A}$$

From $v_o = -Av_1 + \frac{1+A}{1+B} v_2$, we conclude that

$$v_o = 0 + \frac{1+A}{1+\frac{1}{A}} v_i = A v_i .$$

Therefore, $\frac{v_o}{v_i} = A = \frac{R_2}{R_1}$.

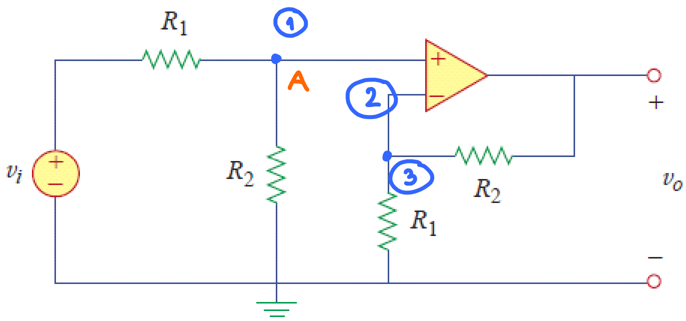
It is also possible to derive the answer from scratch.



1 By the voltage divider formula (which is applicable because $i_+ = 0$), we have

$$v_+ = \frac{R_2}{R_1 + R_2} v_i$$

(Alternatively, we can apply KCL at A to get



(which is applicable because $i_+ = 0$), we have

$$v_+ = \frac{R_2}{R_1 + R_2} v_i$$

(Alternatively, we can apply KCL at A to get

$$\frac{v_+ - v_i}{R_1} + \frac{v_+ - 0}{R_2} + \underset{\underset{i_+}{\uparrow}}{0} = 0 \, .$$
)

2 By rule #2 of ideal Op Amp, $v_- = v_+ = \frac{R_2}{R_1+R_2} v_i$

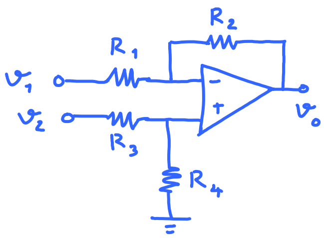
3 Apply KCL at the inverting terminal of the op amp.

$$\frac{v_- - v_o}{R_2} + \frac{v_-}{R_1} = 0 \Rightarrow v_o = R_2 \left(\frac{1}{R_1} + \frac{1}{R_2} \right) v_-$$

$$= R_2 \times \frac{\cancel{R_1 + R_2}}{R_1 \cancel{R_2}} \times \frac{\cancel{R_2}}{\cancel{R_1 + R_2}} v_i = \frac{R_2}{R_1} v_i$$

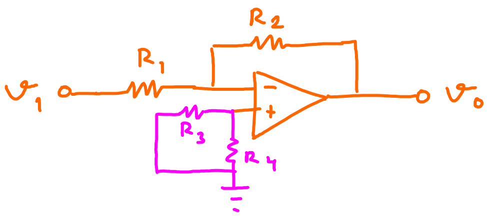
Alternatively, we can use the voltage divider formula (which is applicable because $i_- = 0$):

$$v_- = \frac{R_1}{R_1 + R_2} v_o \, .$$



To apply superposition theorem, we activate one source at a time.

v_1 acts alone



$i_+ = 0 \Rightarrow$ No current through $R_3 // R_4$

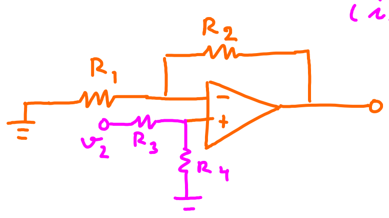
$\Rightarrow v_+ = 0$

Inverting Amplifier: $v_o = -\frac{R_2}{R_1} v_1 = -A v_1$

v_2 acts alone

Voltage divider: $v_+ = \frac{R_4}{R_3 + R_4} v_2$

$(i_+ = 0)$



$v_o = (1 + A) \times \frac{R_4}{R_3 + R_4} v_2$

$= \frac{1+A}{1+B} v_2$

Non-inverting Amplifier

Combine the answers : $v_o = -A v_1 + \frac{1+A}{1+B} v_2$

Q5 [Alexander and Sadiku, 2009, Q5.57]

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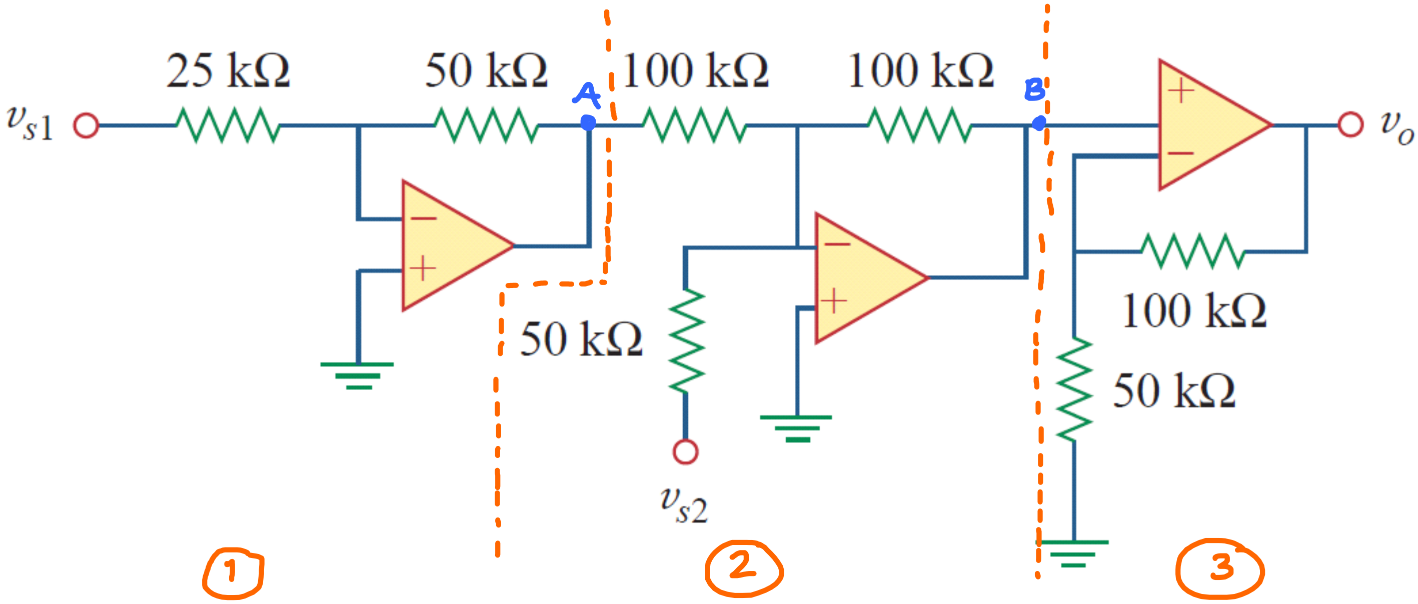


There are three stages.

Observe that stage 1 is simply an inverting amplifier.

So, $v_A = -\dfrac{50\,k\Omega}{25\,k\Omega} \times v_{s_1} = -2\,v_{s_1}$

Observe that stage 2 is simply a summing amplifier.

So, $v_B = -\left(\dfrac{100\,k\Omega}{100\,k\Omega} v_A + \dfrac{100\,k\Omega}{50\,k\Omega} v_{s_2}\right)$

$= -(v_A + 2\,v_{s_2}) = -(-2\,v_{s_1} + 2\,v_{s_2})$

$= 2\,v_{s_1} - 2\,v_{s_2}$

Observe that stage 3 is simply a non-inverting amplifier.

So, $v_o = \left(1 + \dfrac{100\,k\Omega}{50\,k\Omega}\right) v_B = 3\,v_B = 6\,v_{s_1} - 6\,v_{s_2}$

Q6 [Alexander and Sadiku, 2009, Q5.49]

Tuesday, July 16, 2013 8:27 PM

We want $v_o = 2(v_2 - v_1) = -2v_1 + 2v_2$

Remark: This problem is similar to an example we did in class.
There we wanted $v_o = 3v_2 - 5v_1$.
We saw two methods that can implement such expression:
1) using one op amp in the form of difference amp.
2) using two op amps by two stages (inverting amp followed by summing amp.

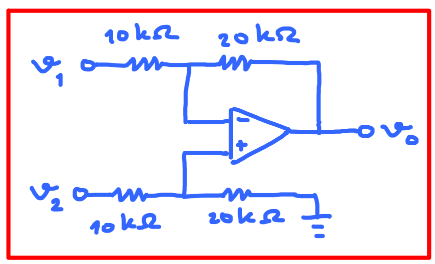
(a) Compare the expression with $v_o = -Av_1 + \frac{1+A}{1+B}v_2$. We see that difference amp can be used to implement such expression by setting

$$A = 2 \quad \text{and} \quad \frac{1+A}{1+B} = 2.$$

With $A=2$, the second requirement is $B = \frac{1}{2}$.
So, we must have

$$\frac{R_2}{R_1} = 2 \quad \text{and} \quad \frac{R_3}{R_4} = \frac{1}{2}.$$

For example, we can set $R_1 = R_3 = 10\ k\Omega$ and $R_2 = R_4 = 20\ k\Omega$ as shown in the figure below:



Alternatively, the lecture note also mentions an intermediate special case of the difference amp. By having $B = 1/A$, we have

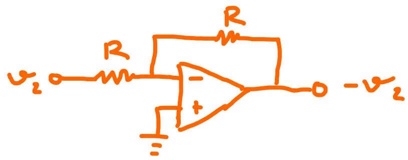
$$v_o = -\underset{R_2/R_1}{A} v_1 + \frac{1+A}{1+\underset{R_3/R_4}{B}} v_2 = -Av_1 + \frac{1+A}{1+\frac{1}{A}} v_2 = A(v_2 - v_1).$$

We again need $A = \frac{R_2}{R_1} = 2$ and $B = \frac{1}{A} = \frac{1}{2}$

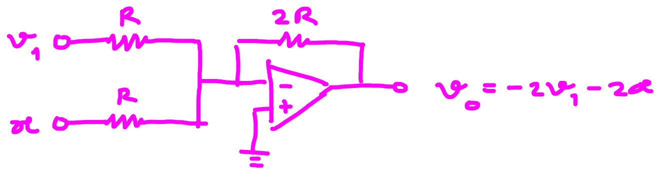
(b) With two op amps, we may divide the task into two stages:

$$v_o = 2(v_2 - v_1) = 2v_2 - 2v_1 = -2(-v_2) - 2v_1$$

The first stage is an inverting amplifier which converts v_2 to $-v_2$.



The second stage is a summing amplifier which convert two inputs, v_1 and x, into $-2v_1 - 2x = -(2v_1 + 2x)$



Here, when we connect the two stages, we have $x = -v_2$;

$$\text{so, } v_o = -2v_1 - 2(-v_2)$$
$$= 2(v_2 - v_1)$$

In the picture below, we choose $R = 10\,k\Omega$.